



Analysis of Junior High School Students' Learning Obstacles on Polyhedron Material Reviewed from the Pirie–Kieren Theory

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ABSTRACT

This study is motivated by the low level of junior high school students' understanding of three-dimensional geometry (solid shapes) and the need for a theoretical framework capable of explaining the dynamics of conceptual development more comprehensively. This research aims to analyze learning obstacles and the growth trajectory of students' understanding using the Pirie–Kieren theory. The study employed a qualitative approach involving diagnostic tests, clinical interviews, observations, and analyses of student learning artifacts, validated through source triangulation, methodological triangulation, and expert confirmation. The findings show that students' understanding develops non-linearly, with most students remaining at the Property Noticing level, characterized by difficulties in 2D–3D visualizations, conceptual reasoning, and multimodal coordination. The phenomenon of folding back consistently appears as a mechanism for reconstructing understanding when students encounter representational or conceptual conflicts. The analysis indicates the predominance of representational, epistemological, and didactic obstacles that hinder students' transition toward formal understanding. In conclusion, the Pirie–Kieren theory effectively maps the development of students' understanding and reveals the root causes of learning barriers in geometry. The study's implications highlight the importance of representation-based learning design, reasoning scaffolding, and facilitating productive folding back to support deeper geometric understanding.

1. Introduction

Geometry is one of the main foundations in the junior high school mathematics curriculum because it plays a role in developing visual–spatial skills, representation, and reasoning that are essential for advanced mathematics learning. One of the important topics at this stage is polyhedra, which require students to connect three-dimensional objects with their two-dimensional representations,

construct nets, understand the properties of solids, and perform calculations involving volume and surface area. However, studies show that geometry particularly three-dimensional shapes remains one of the most difficult topics for students. These difficulties arise in imagining three-dimensional forms, connecting nets to the actual solids, and generalizing concepts. This phenomenon is not only local but also consistent across various international studies, which reveal that geometric understanding develops gradually and requires appropriate representational support.

Field observations show that junior high school students often experience learning obstacles, whether ontogenetic, epistemological, or didactical. Ontogenetic obstacles emerge when students' cognitive readiness for three-dimensional concepts is inadequate; epistemological obstacles occur when new concepts do not align with prior learning experiences; while didactical obstacles arise from instructional approaches that do not adequately support visual exploration and manipulative activities. These findings are in line with Hakim & Murtafiah (2022), who show that learning obstacles in sequences and series can be clearly mapped through the dynamics of mathematical understanding. Similarly, Nopa et al. (2019) reveal that students' understanding in problem-solving remains at the early layers of understanding based on the Pirie–Kieren theory, making it difficult for them to reach the stages of generalization and formalization. Various studies also highlight representational difficulties in other topics such as exponents Syafiqoh et al. (2018), ratios Arenas-Peñaloza et al. (2024), series, linear functions, circles, and geometric reflections Hidayah et al. (2024), consistently showing that students' understanding develops gradually and is influenced by the structure of prior concepts.

To gain deeper insight into these obstacles, this study employs the Pirie–Kieren theory of mathematical understanding development introduced by Pirie & Kieren (1994). The theory describes understanding as a layered, dynamic process that progresses through eight levels: Primitive Knowing, Image Making, Image Having, Property Noticing, Formalising, Observing, Structuring, and Inventising. This process is nonlinear and allows for folding back, which is when students return to earlier layers to strengthen understanding before moving forward. This concept has been examined extensively by Irvine (2023) and Martin & Towers (2016), who assert that folding back is a natural mechanism when students encounter conceptual obstacles. In the context of dynamic learning environments, Gulkilik et al. (2020) show that understanding can be accelerated through multi-representational environments and dynamic geometry technology, although it still depends on the quality of teacher intervention (Yao & Manouchehri, 2022).

The application of the Pirie–Kieren theory across various mathematical domains provides evidence that this model is highly sensitive in identifying critical points in understanding development. For instance, Khairunnisa et al. (2019) found that students' understanding of rectangles varies across student types at each level of the Pirie–Kieren framework, demonstrating that layered analysis can reveal differences in understanding that conventional tests may overlook. In other concepts, Sengul & Argat (2015) demonstrate how the theory can map students' understanding of factorials and rational numbers, including the cognitive structures

that shape their conceptual mind maps. Research by Ramírez & Cano (2013) also shows that the dynamics of mathematical understanding, including in concepts such as limits and partitive division, develop through shifts between levels that do not always align with the order of content in the formal curriculum.

In addition, research by Meel (2003), which compares the Pirie–Kieren model with the APOE theory, provides epistemological arguments that the Pirie–Kieren model offers a richer perspective for describing the evolution of mathematical understanding. Hakim (2019) in the context of mathematical proof also emphasize that the theory helps identify how students transition from procedural understanding to formal deductive understanding. Zawawi et al. (2023) reinforce these findings by showing that students' understanding can be enhanced through appropriate folding back strategies when facing cognitive obstacles.

In the school learning context, Runesson (2016) asserts that learning theory plays an important role in designing lesson studies, including the use of the Pirie–Kieren theory to identify variations in student understanding. Meanwhile, research by Zaqya (2024) highlights the relevance of this theory within the Merdeka Belajar curriculum, which places the development of understanding as the central focus of learning. Thus, analyzing junior high school students' learning obstacles in polyhedra through the lens of the Pirie–Kieren theory is not only theoretically relevant but also practically significant, as it provides a comprehensive picture of how students progress or become stuck at certain levels when understanding three-dimensional objects.

Although research related to the Pirie–Kieren theory has grown extensively, studies that specifically analyze learning obstacles in polyhedra remain limited. In fact, the characteristics of this topic demand complex visualization skills, the use of multiple representations, and the formal interpretation of geometric properties. This gap underscores the need for in-depth research that maps the dynamics of students' understanding in the context of three-dimensional geometry. Therefore, this study contributes to filling this gap by systematically analyzing learning obstacles through the Pirie–Kieren theoretical framework to produce an understanding map that can serve as the basis for innovations in geometry instruction.

2. Methodology

This study employed a descriptive qualitative approach focusing on deeply exploring junior high school students' learning obstacles in the topic of three-dimensional shapes with flat surfaces through the theoretical framework of the Pirie–Kieren model. This approach was selected because the characteristics of geometric understanding are dynamic, layered, and often involve folding-back mechanisms when students encounter difficulties. In line with previous studies that examined the development of mathematical understanding through analyses of representations, thinking dynamics, and error patterns (Khairunnisa et al., 2019) and (Yao & Manouchehri, 2022), this design enables an in-depth exploration of students' cognitive structures as well as epistemological, ontogenic, and didactical

obstacles. The research subjects were eighth-grade students from a junior high school in District X, selected through purposive sampling based on variations in initial ability. This selection follows previous research practices emphasizing the importance of choosing representative subjects to reveal the dynamics of understanding in the contexts of geometry and arithmetic.

Instrument

The primary research instrument in this study was the researcher, supported by several data collection tools, including a diagnostic geometry test, clinical interview guidelines, and observation sheets. The diagnostic test was designed to map students' levels of understanding based on the Pirie–Kieren framework, ranging from Primitive Knowing to Inventising. The clinical interview guidelines were developed to explore students' thinking processes, representation strategies, and indications of folding-back mechanisms, following the approaches of Martin & Towers (2016), and Nurhidayah et al. (2026). Observation sheets were used to systematically document students' written work and behavioral responses during problem-solving activities, enabling the identification of transitions between levels of understanding and emerging patterns of learning obstacles, as suggested by Runesson (2016).

Data Collection

Data were collected through three main techniques. First, a diagnostic geometry test was administered to identify students' initial understanding levels and to detect potential learning obstacles. Second, clinical interviews were conducted to gain deeper insights into students' cognitive processes, particularly in terms of how they construct, modify, and reorganize their understanding when facing difficulties. Third, direct observation of students' work was carried out to capture real-time shifts in understanding levels and the occurrence of folding-back mechanisms. The combination of these techniques allowed for comprehensive and complementary data collection, ensuring that both cognitive processes and observable behaviors were thoroughly examined.

Data Analysis

Data analysis was conducted through stages of data reduction, coding, and interpretation using the Pirie–Kieren framework. Students' understanding was categorized into eight hierarchical levels, while learning obstacles were identified based on recurring patterns of conceptual and representational errors. The analysis also focused on identifying folding-back mechanisms to understand how students revisit previous levels of understanding when encountering difficulties, as highlighted by Yumna & Maksum (2025). To ensure data validity, this study employed source triangulation (tests, interviews, and observations), method triangulation, member checking, and expert review. These procedures ensured consistency between empirical findings and theoretical interpretation, and are aligned with validation practices in studies of geometry learning and mathematical understanding in various international contexts.

3. Results and Discussion

The exploration results show that the selected eighth-grade class has sufficient diversity in cognitive and representational abilities to be analyzed using the Pirie–Kieren framework. Initial observations revealed variations in visualization strategies and levels of understanding of 3D shapes, making purposive sampling the appropriate approach to selecting subjects capable of demonstrating different understanding trajectories. The selected students reflect a spectrum of understanding levels, enabling an in-depth analysis of folding back processes, developmental dynamics, and emerging epistemological and ontogenic obstacles, as recommended in contemporary Pirie–Kieren studies. Thus, this careful selection ensures that the study can present a comprehensive description of the variations in students’ understanding of three-dimensional geometric shapes.

The administration of the diagnostic test was the initial stage to identify learning obstacles, representation errors, and students’ understanding levels on the topic of three-dimensional shapes with flat surfaces. The test was designed based on the eight layers of understanding from the Pirie–Kieren theory, enabling the tracing of how students construct, represent, and gradually develop geometric concepts. The test results indicated a wide variation in understanding, ranging from students still at the Primitive Knowing level to those approaching the Formalising level. This pattern aligns with previous research findings, such as those by, Iwan et al. (2026), which reported discontinuities in representation when students engage with three-dimensional objects. Similar difficulties were also documented in international geometry studies, particularly related to visualization, net construction, and generalization of shape properties, as shown in Table 1.

Table 1. Brief Analysis of Diagnostic Test Results

Diagnostic Aspect	Main Student Findings	Level (Pirie–Kieren)	Indicated Obstacles
Identification of shapes and basic properties of 3D solids	Students recognize the shapes but do not yet understand their properties	Primitive Knowing – Image Making	Ontogenic obstacles in understanding basic concepts
Visualization of nets and rotation of solids	Errors in determining corresponding faces; inaccurate nets	Image Having – Property Noticing	Representational obstacles and spatial misconceptions
Connecting 2D–3D representations	Students fail to relate flat images to actual solids	Property Noticing – Formalising	Epistemological obstacles in coordinating representations
Calculating surface area and volume	Errors in substituting elements; incorrect formula selection	Image Having – Formalising	Procedural and conceptual obstacles
Generalizing patterns and relations among solids	Only some students can explain mathematical reasoning	Formalising – Structuring	Limited generalization ability
Indications of folding back mechanism	Students redraw shapes to understand properties	Transitions between levels	Folding back due to unstable understanding

The overall analysis of the diagnostic test results indicates that most students have not reached consistent understanding at the Property Noticing and Formalising levels, making representational errors and spatial misconceptions the dominant obstacles. The finding that students often return to earlier stages such as redrawing solids to verify their understanding indicates the occurrence of folding back, consistent with the characteristics of mathematical understanding described in (Sanjaya et al., 2019). These results reinforce the idea that geometric understanding does not develop linearly; instead, it requires diagnostic tasks capable of revealing students' cognitive trajectories in depth. Overall, the diagnostic test provides a strong basis for identifying subjects for further analysis and designing process-based clinical interviews.

This tendency of students to remain at the Property Noticing level can be causally explained by the lack of integration between visual representation and formal reasoning structures. At this stage, students are able to recognize geometric properties but fail to construct stable conceptual relationships that support generalization. This indicates a disruption in the transition from Image Having to Formalising, where understanding should evolve into structured mathematical reasoning. This stagnation is primarily influenced by epistemological obstacles, particularly incomplete prerequisite knowledge and weak connections between two-dimensional and three-dimensional representations. As a result, students rely on surface-level recognition rather than deeper conceptual organization, preventing progression to higher levels of understanding.

The initial analysis was conducted after the diagnostic results were mapped into the eight layers of the Pirie–Kieren model. The main purpose of this stage was to identify student understanding comprehensively, including representation patterns, types of errors, and indications of folding back. The findings show that students have diverse understanding trajectories, requiring purposive selection of cases for deeper analysis through clinical interviews and follow-up observations. This selection approach aligns with previous Pirie–Kieren studi Mutmainnah et al. (2025), which emphasize the importance of diverse understanding profiles as a basis for exploring thinking processes. Similar findings were also noted in studies by Gulkilik et al. (2020), showing that initial ability differences strongly influence the dynamics of geometric understanding and level transitions.

Clinical interviews were conducted to examine in detail students' thinking processes when solving 3D geometry tasks, especially polyhedra. The interviews were designed to elicit students' reasoning, representations, and strategies naturally, allowing the researcher to identify the dynamics of transitions between Pirie–Kieren levels and map folding back mechanisms when students encountered obstacles. This approach is consistent with Pirie–Kieren research methodology in studies by Martin & Towers (2016), which highlight the importance of verbal and representational exploration for understanding the development of mathematical understanding. Interview results indicate that students use various strategies but often experience conceptual instability that forces them to return to basic representations, a pattern also found in Hakim & Murtafiah (2022), as shown in Table 2.

Table 2. Brief Analysis of In-Depth Clinical Interview Results

Analyzed Aspect	Main Findings on Students' Thinking	Identified Pirie–Kieren Level	Indication of Folding Back / Obstacle
2D–3D Representation	Students repeatedly redraw shapes to confirm corresponding faces	Image Making → Image Having	Folding back due to unstable visualization
Analysis of Solid Properties	Students understand some properties but cannot explain formal reasoning	Image Having → Property Noticing	Epistemological obstacles; lack of concept consolidation
Surface Area Problem Solving	Students obtain correct calculations but do not understand formula choice	Property Noticing → Formalising	Unstable transition; folding back to procedural
Argumentation and Generalization	Students connect several concepts but lose track when explaining patterns	Formalising → Structuring	Folding back due to weak idea organization
Coordination of Multimodal Representations	Students use gesture, drawings, and verbal explanations in limited ways	Moving between Image Having – Property Noticing	Multimodal representational obstacles
Metacognition and Strategy Control	Some students say “I’m not sure,” then restart procedures from the beginning	Image Making → Primitive Knowing	Extreme folding back; indicator of unstable understanding

Interview results show that students’ understanding dynamics are heavily influenced by how they represent and verify geometric concepts. Folding back mechanisms appear prominently, particularly in visualization and property analysis tasks, indicating that students need more explicit reinforcement of visual–symbolic representations. The data also reveal gaps between procedural and conceptual abilities, consistent with international research in geometry and algebra. Thus, clinical interviews provide rich insights into how students navigate their understanding trajectories, reinforcing the relevance of the Pirie–Kieren model in analyzing complex and non-linear mathematical thinking processes.

Conceptually, the occurrence of folding back in this study is triggered by cognitive conflict arising from inconsistencies between students’ visual representations and emerging formal reasoning. When students cannot reconcile these forms, they return to earlier layers, such as redrawing shapes or reinterpreting basic properties, to stabilize their thinking. However, this folding back is often unproductive, as students repeatedly revisit earlier stages without achieving conceptual refinement. This indicates that the learning environment does not sufficiently support the transformation of revisited understanding into higher-level reasoning. Observations of problem-solving processes were conducted to capture students’ cognitive behavior during interaction with polyhedron tasks. The observations focused on visual, symbolic, and verbal strategies used by students, as well as recurring difficulties in coordinating 2D–3D representations, analyzing solid properties, and performing geometric calculations. This approach aligns with the view that mathematical understanding develops dynamically and multimodally, as described by Pirie & Kieren (1994). Results show substantial variation in student strategies

along with consistent obstacles, particularly in spatial representation coordination and concept consolidation, as shown in Table 3.

Table 3. Brief Analysis of Problem-Solving Observation Results

Observation Aspect	Main Student Behaviors	Related Pirie–Kieren Level	Emerging Obstacles
Visual Strategies	Students frequently redraw shapes and nets to verify forms	Image Making – Image Having	Unstable visualization; folding back
Coordination of 2D–3D Representations	Difficulty connecting nets to actual solids	Image Having – Property Noticing	Representational and spatial obstacles
Symbolic Strategies	Students follow procedures but do not understand the meaning of formulas	Property Noticing – Formalising	Procedural–conceptual obstacles
Verbal Argumentation	Explanations remain descriptive, not yet mathematical justifications	Property Noticing – Formalising	Weak reasoning and generalization
Use of Gestures and Manipulatives	Gestures help visualize rotations but often lack consistency	Image Making – Property Noticing	Multimodal coordination obstacles
Indications of Folding Back in Complex Tasks	Students return to drawing when confused about properties	Transition between levels	Folding back as a stabilization mechanism

Observations revealed that students' problem-solving processes were strongly influenced by their ability to coordinate multiple forms of representation. When visual representations became unstable, students tended to experience folding back to earlier stages to reorganize their understanding a dynamic consistent with findings from international Pirie–Kieren studies Martin & Towers (2016). Representational obstacles appeared as the most dominant pattern, especially in tasks requiring transformations from nets to three-dimensional figures. In addition, some students demonstrated procedural understanding without conceptual support, reinforcing the gap between symbolic ability and mathematical reasoning. Thus, these observations provide strong evidence that students' geometry understanding does not develop linearly and is highly dependent on the quality of representational strategies they employ.

This inconsistency among visual, symbolic, and verbal representations indicates a failure in representational coordination, which is essential for advancing to higher levels of understanding. Within the Pirie–Kieren framework, this coordination is required to move from Property Noticing to Formalising. Without such integration, students remain dependent on fragmented representations, limiting their ability to construct generalizable mathematical concepts. Learning artifacts were collected to strengthen the findings from diagnostic tests, clinical interviews, and observations of the problem-solving process. The artifacts consisted of audio recordings of interviews, field notes on students' behaviors and strategies, as well as student work such as drawings, nets, written verbal explanations, and calculations. This documentative approach enabled the researcher to trace students' thinking more

holistically and triangulatively, in line with recommendations from Pirie–Kieren research, which emphasizes the importance of multimodal analysis as a reflection of the development of mathematical understanding (Martin & Towers, 2016; Pirie & Kieren, 1994). These artifacts also provided concrete evidence of representational dynamics, conceptual instability, and folding back mechanisms that frequently appear in geometry and mathematical representation studies, as shown in Table 4.

Table 4. Summary Analysis of Learning Artifact Documentation

Type of Artifact	Key Findings from Artifact Analysis	Indicated Pirie–Kieren Level of Understanding	Patterns of Difficulties or Dynamics
Written Work (drawings, nets, calculations)	Many drawing revisions; calculations correct but not consistent with representations	Image Making – Property Noticing	Instability of visualization, conceptual misconceptions
Observation Field Notes	Gestures used to visualize rotation; students return to earlier steps	Image Having – Property Noticing	Strong indication of folding back
Interview Recordings	Verbal explanations repeated and changed when asked to justify reasoning	Property Noticing – Formalising	Difficulties in reasoning and coordinating representations
Students' Explanation Drafts	Descriptions are procedural without conceptual generalization	Formalising – Structuring	Procedural–conceptual gap
Sketches or Manual Revisions	Repeated re-sketching as a form of verification	Image Making – Image Having	Folding back for stabilizing understanding

The analysis of learning artifacts showed that students tended to leave cognitive traces consistent with the developmental dynamics described in the Pirie–Kieren theory. Repeated revisions of drawings, the use of gestures to refine visualization, and fluctuating verbal explanations indicated that students often returned to previous layers of understanding (folding back) to reorganize concepts a pattern widely documented in geometry and mathematical representation studies (Irvine, 2023). The artifacts also confirmed gaps between procedural and conceptual abilities and weak coordination between visual–symbolic representations, as reported in studies by Syafiqoh et al., (2018). Thus, the documentation of learning artifacts played an important role in data triangulation and provided strong evidence of the complexity of students' understanding trajectories in polyhedron topics. The repeated revisions found in students' work indicate epistemological instability, where students continuously reconstruct their understanding due to the absence of firmly established concepts.

This suggests that students do not possess stable internal representations, forcing them to rely on repeated external reconstruction. This condition reinforces the idea that understanding development is recursive rather than linear, with students oscillating between levels due to unresolved conceptual conflicts. A synthesis of findings was conducted by integrating all data diagnostic tests, clinical interviews,

observations, and artifacts to construct a comprehensive picture of students' understanding development on polyhedron topics. This synthesis revealed major patterns of understanding trajectories, types of epistemological, ontogenic, and didactic obstacles, and folding back dynamics that emerged as students attempted to stabilize their understanding. The integrative approach aligns with the theoretical framework of (Jannah et al., 2022) and analytical developments in mathematical representation studies (Irvine, 2023; Ramírez & Cano, 2013). This synthesis also offers important implications for pedagogical design, especially regarding the need to strengthen visual–symbolic representations and use diagnostic tasks that reveal developmental trajectories, as shown in Table 5.

Table 5. Summary Analysis of Synthesis and Findings

Synthesis Area	Integrated Findings	Theoretical Meaning (Pirie–Kieren)	Pedagogical Implications
Students' Understanding Trajectory	Development is non-linear, predominantly stops at Property Noticing	Understanding moves back and forth across levels (folding back)	Need for layered instructional design (layered tasks)
Visual–Spatial Representation	Difficulties connecting 2D–3D and nets	Weakness in Image Making–Image Having	Use of concrete models and dynamic simulations
Conceptual Reasoning	Limited verbal explanations; fragmented reasoning	Weakness in Property Noticing–Formalising	Argumentation exercises and reasoning scaffolding
Procedural Application	Students rely on formulas without conceptual understanding	Gap between representation and procedure	Need integration of representation–procedure
Epistemological Obstacles	Unstable prerequisite concepts	Hinders transition to higher levels	Strengthening basic concepts through reconstructive learning
Didactical Obstacles	Lack of manipulative and multimodal support	Obstacle in coordinating representations	Use of visual media and task-based learning
Folding Back Phenomenon	Students often return to initial drawings when confused	Mechanism for stabilizing understanding	Teachers need to facilitate productive folding back

The synthesis indicates that students' understanding of geometry does not progress in a linear accumulation of knowledge, but rather follows complex and recursive trajectories characterized by forward movement and backward recursion across the Pirie–Kieren levels. However, a more critical interpretation suggests that students' stagnation at certain levels particularly at Property Noticing is not merely a cognitive limitation, but is structurally related to the fragility of conceptual coordination between perceptual experience and formal abstraction. In other words, students do not simply “fail to progress,” but remain trapped in representational dependence, where visual impressions dominate over relational and structural reasoning. This condition explains why transitions toward Formalising are consistently interrupted.

Representational and epistemological obstacles are not independent categories, but mutually reinforcing constraints that stabilize students at intermediate levels of understanding. Representational difficulty in coordinating 2D and 3D structures weakens the epistemological foundation needed to construct mathematical relations among edges, faces, and diagonals. Consequently, students are unable to establish invariant conceptual structures required for abstraction. This condition explains why students often recognize properties of solids without being able to justify them mathematically. The findings extend previous research by Gulkilik et al. (2020), which identified spatial coordination difficulties, by showing that such difficulties do not merely hinder comprehension, but actively regulate the boundary of conceptual progression within the Pirie–Kieren hierarchy.

A deeper conceptual explanation of the observed stagnation suggests that the transition from Property Noticing to Formalising requires a reorganization of cognitive structures from perceptually anchored reasoning to relational abstraction. This transition is cognitively demanding because it requires students to detach meaning from physical representation and reconstruct it through internalized mathematical relations. When this restructuring fails, students remain in a stable but limited interpretive system. This aligns with Khairunnisa et al. (2019), yet the present study extends their findings by showing that epistemological gaps do not only cause errors but also determine the “ceiling” of developmental progression.

The mechanism of folding back, in this context, should not be interpreted as a simple return to prior knowledge, but as a reconfiguration process triggered by cognitive dissonance between representation and abstraction. Folding back occurs when students encounter breakdowns in meaning coherence, particularly when their visual interpretations cannot support formal explanation. During this process, students revisit earlier levels not as regression, but as a strategy to reconstruct conceptual stability. This supports Martin & Towers (2016), who conceptualize folding back as productive reconstruction rather than failure, but the present findings further elaborate that folding back becomes persistent when representational coordination remains unresolved, thereby creating cyclical rather than progressive movement.

The study further reveals that folding back is often initiated by representational instability, particularly when students attempt to translate between nets and three-dimensional solids. This instability generates epistemological conflict, forcing students to reconstruct meaning through repeated reinterpretation of earlier concepts. In this sense, folding back functions as a diagnostic indicator of conceptual fragility rather than merely a developmental phase. This finding extends by Irvine (2023) specifying that folding back is not uniformly distributed across learning processes, but is concentrated at structurally sensitive transition points within the Pirie–Kieren model. Moreover, multimodal representation analysis demonstrates that drawings, gestures, and verbal explanations do not always operate in coordination; instead, they often reveal epistemic fragmentation. This fragmentation indicates that representation is not simply a learning aid but a site of conceptual instability. When modalities conflict, students tend to privilege visual intuition over formal reasoning, reinforcing partial understanding. This finding

supports Arenas-Peñaloza et al. (2024), but also extends their argument by showing that representational inconsistency is not only symptomatic but also generative of learning stagnation.

The dominance of procedural instruction further intensifies this condition. Procedural fluency without conceptual anchoring produces what can be interpreted as “surface-level competence,” where students can execute formulas without understanding their structural origin. This explains why procedural success does not translate into conceptual advancement, as also noted by Rexhepi & Makasevska (2024). However, this study adds a critical dimension: procedural instruction may inadvertently suppress folding forward by reducing opportunities for conceptual restructuring, thereby increasing reliance on folding back cycles. From a theoretical standpoint, these findings strengthen the explanatory power of the Pirie–Kieren model by demonstrating that level transitions are not merely developmental stages but contingent processes shaped by representational and epistemological stability. In contrast to linear developmental assumptions, this study shows that progression depends on the resolution of internal contradictions between perception and abstraction. This contributes to the argument of Nopa et al. (2019) by emphasizing that mathematical understanding is not only layered but dynamically unstable, especially in spatial geometry learning contexts.

Pedagogically, the implication is not simply the use of visual aids, but the deliberate orchestration of representational conflict to stimulate productive folding back. Instruction must therefore be designed not only to reduce difficulty, but to strategically manage cognitive tension so that students can reconstruct meaning across levels. The use of manipulatives, dynamic geometry environments, and guided abstraction scaffolds, as suggested by Hakim & Murtafiah (2022), becomes more critical when interpreted through this lens, because they function as mediators for stabilizing transitions rather than merely supporting visualization.

Overall, this study positions students’ stagnation at Property Noticing not as a deficit, but as a structural outcome of unresolved representational and epistemological disjunctions. In comparison with previous studies, this research not only confirms the existence of learning obstacles but also extends them by explaining how these obstacles actively shape the trajectory of understanding within the Pirie–Kieren framework. Thus, the contribution of this study is both theoretical by refining the interpretation of folding back as a mechanism of conceptual reconstruction under instability and pedagogical by emphasizing that meaningful learning progression in geometry depends on managing transitions rather than simply delivering content.

4. Conclusion

This study concludes that junior high school students’ understanding of three-dimensional geometric shapes develops in a non-linear manner and is predominantly constrained by representational and epistemological obstacles, as well as weak multimodal coordination. Most students remain at the Property

Noticing level within the Pirie–Kieren theory, indicating limited ability to construct coherent mathematical reasoning and to generalize concepts consistently. In relation to the research objective, this study successfully identifies and analyzes the types of learning obstacles and the dynamics of understanding development experienced by students, particularly in terms of difficulties in coordinating 2D–3D representations, conceptual reasoning, and procedural application. The findings also reveal that the phenomenon of folding back consistently emerges as a cognitive mechanism for reconstructing understanding when students encounter conceptual conflicts, especially in visualization and explanation tasks.

Furthermore, data triangulation confirms that unstable visualization, inaccuracies in conceptual reasoning, and the dominance of procedural learning are the main factors that hinder students' progression to higher levels of understanding. These results not only reaffirm the relevance of the Pirie–Kieren theory as an analytical framework, but also extend previous studies by demonstrating that the critical barrier in geometry learning lies in the transition between understanding levels, particularly from Property Noticing to Formalising. Therefore, this study contributes theoretically by providing a deeper explanation of how representational instability and folding back interact in shaping students' understanding trajectories, and practically by emphasizing the need for instructional designs that support representational integration, conceptual reasoning, and productive folding back. Such approaches are essential for fostering deeper and more sustainable understanding of three-dimensional geometric concepts.

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